

contents of his *Gesammelte Abhandlungen*. Moreover, it is well known that Lie's interest in continuous groups grew out of an intense preoccupation with a new geometrical approach to the theory of ordinary and partial differential equations.

These weaknesses reflect many of the standard problems that arise when mathematicians undertake historical studies of their discipline. In the present case, the author clearly has a solid grasp of the mathematics under discussion and considerable insight into the modern developments that have grown up out of them. What is lacking here, however, is historical sensibility, and without that the history of mathematics can never be more than a playground for anecdotes, tall tales, and a fundamentally ahistorical interest in mathematics as a collection of disembodied ideas.

Riemann, Topology, and Physics. By Michael Monastyrsky. Boston, Basel, Stuttgart (Birkhäuser). 1987. xiii + 158 pp.

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Universally acknowledged as one of the greatest mathematicians of the nineteenth century, Riemann has become something of a household word for the working mathematician. The influence he exerted on many branches of mathematics is apparent in the terminology—Riemann surfaces, Riemannian manifolds, the Cauchy–Riemann equations, the Riemann integral, the Riemann zeta function, the Riemann–Roch theorem, and so on. The evident questions to ask are those asked of any great thinker—who was he? what kind of life did he lead? how did he come upon his great ideas? what has been the fate of his work?

Monastyrsky's book answers these questions but in a somewhat unusual manner—it is not meant to be a unified text, and offers two essays, different in intention and tone, which are related by certain underlying themes. The first essay is a translation of the first Russian language biography of Riemann, written on the occasion of the 150th anniversary of Riemann's birth. The second treats the modern interaction of physics and mathematics; in particular, those topics where topology is applied. No effort has been made to bring the essays together, but the wisdom of the publisher is still perceived. The body of work left by Riemann points in two directions—the introduction of the underlying geometric and topological structures in analysis, and the application of such global methods to the physical world. Clearly, he would have read the second essay with great relish.

The historical essay on Riemann's life and work is short and well-written. The level of detail and scholarship, however, does not go beyond that found in Bell or in Dedekind's *Lebenslauf* (found in Riemann's collected works). As an introduction to this great thinker and his ideas, the essay serves the casual reader well. The comments on the difficult reception of Riemann's ideas, based on the then un-

proven Dirichlet's Principle and on new geometric (that is, topological) ideas, are careful and important to describing his life as a nineteenth-century mathematician. Monastyrsky closes his essay with a chapter on the fate of Riemann's work after his death. The author's expertise and modern viewpoint highlight Riemann's forward vision and the richness of the mathematics developed around his ideas.

Modern theoretical physics has been changing at a rate that can scarcely be kept up with by its practitioners. Ideas such as string theory and conformal field theory that are in the recent focus of research share a theme that is laid out in the second essay, that is, that the topology of the underlying spaces of interest to physicists is important, and topological invariants of these spaces are physical invariants as well. Monastyrsky introduces the necessary topology to ease the reader into his examples, and then describes physics research on such topics as liquid crystals, gauge field theory, and spontaneous symmetry-breaking. The local symmetries of each theory can be coded up into the objects of interest to topologists, such as fiber bundles, and the global aspects of the topology appear as physically significant phenomena. For example, in the case of liquid crystals, certain singularities must appear on the surface of a region because the underlying vector fields are restricted by the topology of the surface. This allows a classification of the singularities and of their interactions, which are called 'boojums.'

Among the topics treated in the second essay are gauge fields and monopoles. Monastyrsky's choice of these topics coincided with a general return to mathematics by physicists in their efforts to extend physical theories. The work of Atiyah and Donaldson on spaces of monopole-like objects has produced spectacular results on the structure of four-dimensional manifolds. For Riemann, research into physical phenomena often paid back the effort with some new mathematics. The author does not take up this theme, but he does offer an introduction to the sources of these new and exciting developments.

In the forward to this book, Freeman Dyson describes Monastyrsky as a writer who 'sees modern ideas in a perspective that goes all the way to Riemann.' As with Riemann before him, the author's vision into topological physics describes new and deep ideas that have already gone far beyond his report. One of the rewards of participation in the mathematical community is the link it forges with its rich and sometimes prescient past. This book reinforces that good feeling.

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